## Chapter 10 Section 1 - Homework Problems

For problems #1 – 12, find a formula for the $n^{th}$ term in the sequence (starting with $n = 0$); $a_n = $?

<table>
<thead>
<tr>
<th>Problem</th>
<th>Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( \left{ 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots \right} )</td>
</tr>
<tr>
<td>2.</td>
<td>( \left{ 1, -\frac{1}{3}, -\frac{1}{9}, -\frac{1}{27}, -\frac{1}{81}, \ldots \right} )</td>
</tr>
<tr>
<td>3.</td>
<td>( \left{ \frac{1}{5}, \frac{16}{25}, \frac{64}{125}, \frac{256}{625}, \ldots \right} )</td>
</tr>
<tr>
<td>4.</td>
<td>( \left{ 0, \frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \frac{8}{81}, \ldots \right} )</td>
</tr>
<tr>
<td>5.</td>
<td>( \left{ 0, -\frac{1}{3}, -\frac{2}{9}, -\frac{4}{27}, -\frac{8}{81}, \ldots \right} )</td>
</tr>
<tr>
<td>6.</td>
<td>( \left{ \frac{1}{7}, \frac{3}{49}, \frac{9}{343}, \frac{27}{2401}, \ldots \right} )</td>
</tr>
<tr>
<td>7.</td>
<td>( \left{ \frac{1}{7}, \frac{-3}{49}, \frac{-9}{343}, \frac{-27}{2401}, \ldots \right} )</td>
</tr>
<tr>
<td>8.</td>
<td>( \left{ \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots \right} )</td>
</tr>
<tr>
<td>9.</td>
<td>( \left{ \frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}, \ldots \right} )</td>
</tr>
<tr>
<td>10.</td>
<td>( \left{ \frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}, \ldots \right} )</td>
</tr>
<tr>
<td>11.</td>
<td>( \left{ \frac{2^2}{3^3}, \frac{4^2}{5^3}, \frac{6^2}{7^3}, \frac{8^2}{9^3}, \ldots \right} )</td>
</tr>
<tr>
<td>12.</td>
<td>( \left{ \frac{2^2}{3^3}, -\frac{4^2}{5^3}, \frac{6^2}{7^3}, -\frac{8^2}{9^3}, \ldots \right} )</td>
</tr>
</tbody>
</table>

13. Find the limit of the sequence featured in homework problem #1.

14. Find the limit of the sequence featured in homework problem #2.

15. Find the limit of the sequence featured in homework problem #3.

16. Find the least upper bound of the sequence featured in homework problem #8.

17. Find the limit of the sequence featured in homework problems #8 and #16.
For problems #18 – 25, find the limit of the given sequence, if it exists.

18. \( a_n = \frac{6n+5}{3n-3} \)

19. \( a_n = \frac{8n+11}{2n-1} \)

20. \( a_n = \frac{5n^2+3n-2}{n^2+9n+1} \)

21. \( a_n = \frac{100n+1}{3n^2+7n+6} \)

22. \( a_n = \frac{2n^3+9n+1}{12-n^2} \)

23. \( a_n = \frac{\ln(n+1)}{n+1} \)

24. \( a_n = \frac{2 + \sin n}{\pi - 2 \tan^{-1} n} \)

25. Suppose that \( \lim_{n \to \infty} a_n = 5 \) and \( \lim_{n \to \infty} b_n = 2 \), and that \( c_n = \frac{2a_n - b_n^2}{a_n^2 + 3b_n} \). Then \( \lim_{n \to \infty} c_n = ? \)

26. Suppose that \( \lim_{n \to \infty} a_n = -3 \) and \( \lim_{n \to \infty} b_n = 4 \), and that \( c_n = \frac{\sqrt{a_n^2 + 2b_n} - 1}{a_n^2 + b_n^2} \). Then \( \lim_{n \to \infty} c_n = ? \)

27. If \( a_n = \frac{(n+2)\pi + \sin(n+2)}{n+2} \), then \( \lim_{n \to \infty} a_n = ? \)

28. If \( a_n = \cos\left(\frac{n^3-5}{n^3}\right) \), then \( \lim_{n \to \infty} a_n = ? \)

29. If \( a_n = \text{sech}\left(\frac{\ln n}{\sqrt{n}}\right) \), then \( \lim_{n \to \infty} a_n = ? \)

30. If \( a_n = 2^n \), then \( \lim_{n \to \infty} a_n = ? \)

31. If \( a_n = \left(-\frac{2}{5}\right)^n \), then \( \lim_{n \to \infty} a_n = ? \)

32. Define a sequence recursively by \[
\begin{align*}
Q_0 &= Q_1 = 2 \\
Q_{n+2} &= \frac{2Q_n + Q_{n+1}}{2}, \text{ for } n \geq 0
\end{align*}
\] Then \( Q_4 = ? \)

33. Define a sequence recursively by \[
\begin{align*}
R_0 &= \sqrt{2} \\
R_{n+1} &= \sqrt{2 + R_n}, \text{ for } n \geq 0
\end{align*}
\] Then \( R_5 = ? \)

*34. Find the limit of the sequence featured in #33.*
35. Define a sequence recursively by

\[ R_0 = 1 \]

\[ R_{n+1} = \frac{1}{1 + R_n}, \quad \text{for} \quad n \geq 0 \]

Then \( R_3 = ? \)

*36. Find the limit of the sequence featured in #35.