For problems #1 – 16, find the sum of the given geometric series.

1. \( \sum_{n=0}^{100} \frac{1}{3^n} = ? \)
2. \( \sum_{n=0}^{88} \frac{7^n}{10^n} = ? \)
3. \( \frac{1}{5^1} - \frac{1}{5^4} + \frac{1}{5^5} - \frac{1}{5^6} + \cdots + \frac{1}{5^{23}} = ? \)

4. \( \frac{-8}{27} + \frac{16}{81} - \frac{32}{243} + \cdots - \frac{2^{19}}{3^{19}} = ? \)
5. \( \frac{1}{2^5} - \frac{1}{2^9} + \frac{1}{2^{13}} - \frac{1}{2^{17}} + \cdots - \frac{1}{2^{45}} = ? \)

6. \( \sum_{n=0}^{\infty} \frac{100}{3^n} = ? \)
7. \( \sum_{n=0}^{\infty} (-1)^n \frac{9^n}{10^n} = ? \)
8. \( \sum_{n=0}^{\infty} e^n = ? \)

9. \( \sum_{n=0}^{\infty} e^{-n} = ? \)
10. \( \sum_{n=0}^{\infty} (-1)^n e^{-n} = ? \)
11. \( \frac{\pi^3}{5^3} - \frac{\pi^3}{5^4} + \frac{\pi^3}{5^5} - \frac{\pi^3}{5^6} + \cdots = ? \)

12. \( \frac{\pi^3}{e^3} - \frac{\pi^3}{e^4} + \frac{\pi^4}{e^5} - \frac{\pi^5}{e^6} + \cdots = ? \)
13. \( \frac{-8}{27} + \frac{16}{81} - \frac{32}{243} + \cdots - \frac{2^{19}}{3^{19}} + \cdots = ? \)

14. \( \frac{3}{2^5} - \frac{3^2}{2^8} + \frac{3^3}{2^{13}} - \frac{3^4}{2^{17}} + \cdots + \frac{3^{11}}{2^{45}} + \cdots = ? \)
15. \( \sum_{n=3}^{\infty} \frac{4^n}{5^n} = ? \)
16. \( \sum_{n=3}^{\infty} \frac{5^n}{4^n} = ? \)

For problems #17 – 20, find a reduced fraction that equals the given repeating decimal number. It’s a good idea to do this using geometric series and also using the traditional short-cut method.

17. \( 0.\overline{17} \)
18. \( 0.\overline{392} \)
19. \( 0.\overline{9} \)
20. \( 0.\overline{7} \)

21. Suppose a ball travels a distance of \( \frac{5}{7} \) meter on its first bounce, and then a distance of \( \frac{5}{7} \) meter on its second bounce, and so on, traveling \( \frac{5}{7} \) of the previous bounce distance on each successive bounce. After an infinite number of bounces, how far will the ball have gone from its starting position?
22. Suppose a ball travels a distance of 9/10 meter on its first bounce, and then a distance of 9/10 meter on its second bounce, and so on, traveling 9/10 of the previous bounce distance on each successive bounce. After an infinite number of bounces, how far will the ball have gone from its starting position?

23. \[ \sum_{k=0}^{\infty} 5^{2k} 7^{-3k} = ? \]
24. \[ \sum_{k=0}^{\infty} 3^{1+2k} 2^{1-k} = ? \]
25. \[ \sum_{k=0}^{\infty} e^{5+2k} \pi^{2-7k} = ? \]
26. \[ \sum_{k=0}^{\infty} e^{4-2k} \pi^{3+2k} = ? \]

27. Suppose that \( \sum_{k=0}^{\infty} a_k = 17 \) and that \( \sum_{k=0}^{\infty} b_k = -5 \). Then \( \sum_{k=0}^{\infty} (2a_k + 3b_k) = ? \)

28. Suppose that \( \sum_{k=0}^{\infty} a_k = -3 \) and that \( \sum_{k=0}^{\infty} b_k = -4 \). Then \( \sum_{k=0}^{\infty} (4a_k - 2b_k) = ? \)

29. \[ \sum_{n=1}^{\infty} \frac{2}{4n^2-1} = ? \] Hint: Do a partial fraction decomposition first; then write out a few terms.

30. \[ \sum_{n=1}^{\infty} \left( \frac{2}{4n^2-1} + \frac{2^{k+1}}{3^{k-1}} \right) = ? \]
31. \[ \sum_{n=3}^{\infty} \left( \frac{4^{n+1}}{5^{n-2}} - \frac{7^{n-2}}{9^{n-3}} \right) = ? \]
32. \[ \sum_{n=0}^{\infty} \left( \frac{3}{1000} + \frac{2}{n+1} \right) = ? \]

33. \[ \sum_{n=2}^{\infty} \left( \frac{e}{3e+n^{-1}} + \frac{n}{n^2+2} \right) = ? \]
34. \[ \sum_{n=0}^{\infty} \tan^{-1}(n) = ? \]
35. \[ \sum_{n=0}^{\infty} \left( \tan^{-1}(n^2) + \frac{e^n}{\pi^n} \right) = ? \]

36. Let \( \kappa = \{c, c, c, \cdots \} \) be a constant sequence. Find \( \Delta \kappa \) and \( \sum \kappa \).

37. Let \( \lambda = \{an\}_{n=0}^{\infty} \) for some constant \( a \). Find \( \Delta \lambda \) and \( \sum \lambda \).

38. Let \( \psi = \{an^2 + bn + c\}_{n=0}^{\infty} \) for some constants \( a \), \( b \), and \( c \). Find \( \Delta^2 \psi = \Delta (\Delta \psi) \).