For problems #1 – 13 find the radius and interval of convergence of the given power series.

1. \( \sum_{n=0}^{\infty} \frac{x^n}{n!} \)
2. \( \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{\pi^n} \)
3. \( \sum_{n=0}^{\infty} \frac{(x-4)^n}{8^n} \)
4. \( \sum_{n=0}^{\infty} \frac{(x+e)^n}{\pi^n} \)

5. \( \sum_{n=0}^{\infty} \frac{x^{2n}}{n!} \)
6. \( \sum_{n=0}^{\infty} \frac{3^{5n} x^{3n}}{n!} \)
7. \( \sum_{n=1}^{\infty} \frac{(nx)^n}{(2n)!} \)
8. \( \sum_{n=1}^{\infty} \frac{(nx)^n}{n!} \)

(Note: Don’t worry about the end-point convergence on #8.)

9. \( \sum_{n=1}^{\infty} \frac{n^{2n} x^n}{n!} \)
10. \( \sum_{n=1}^{\infty} \frac{n^n x^n}{10^n} \)
11. \( \sum_{n=1}^{\infty} \frac{x^n}{n!} \)
12. \( \sum_{n=1}^{\infty} \frac{x^n}{n^4} \)

(Assume that \( A > 1 \))

(Assume that \( A < 1 \))

13. \( \sum_{n=0}^{\infty} \frac{n^3 x^n}{(n+1)^n} \)

14. Suppose that the power series \( \sum_{n=0}^{\infty} a_n (x-1)^n \) is known to converge when \( x = -2 \), but the series diverges when \( x = 5 \). What can we conclude about the convergence of the series at \( x = -4, x = -3, x = -2.5, x = 3, x = 4.5 \), and at \( x = 6 \)?

15. Suppose that the power series \( \sum_{n=0}^{\infty} a_n (x-2)^n \) is known to converge when \( x = -1 \), but the series diverges when \( x = -3 \). What can we conclude about the convergence of the series at \( x = -4, x = -2, x = 0, x = 5, x = 6, x = 7 \), and at \( x = 8 \)?
16. Suppose that the power series \( \sum_{n=0}^{\infty} a_n (x+1)^n \) is known to converge when \( x = -3 \), but the series diverges when \( x = 3 \). What can we conclude about the convergence of the series at 

\( x = -6, \; x = -5, \; x = -4, \; x = 0, \; x = 1, \; x = 2, \) and at \( x = 4 \)?

For problems #17 – 21, find the derivative of the given power series.

17. \( f(x) = \sum_{k=0}^{\infty} \frac{x^k}{k+1} \)

18. \( g(x) = \sum_{k=0}^{\infty} \frac{4^k x^k}{k!} \)

19. \( h(x) = \sum_{k=0}^{\infty} \frac{3^k x^k}{(k!)^2} \)

20. \( P(x) = \sum_{k=0}^{\infty} \frac{2^k x^k}{k! (k^2 + 1)} \)

21. \( \psi(x) = \sum_{k=0}^{\infty} \frac{(k+1)^k x^k}{(k!)^2} \)

For problems #22 – 26, find the integral of the given power series. Let the constant of integration be 0.

22. \( f(x) = \sum_{k=0}^{\infty} \frac{x^k}{k+1} \)

23. \( g(x) = \sum_{k=0}^{\infty} \frac{4^k x^k}{k!} \)

24. \( h(x) = \sum_{k=0}^{\infty} \frac{3^k x^k}{(k!)^2} \)

25. \( P(x) = \sum_{k=0}^{\infty} \frac{2^k x^k}{k! (k^2 + 1)} \)

26. \( \psi(x) = \sum_{k=0}^{\infty} \frac{(k+1)^k x^k}{(k!)^2} \)

For problems #27 – 29, do each of the following:

1. Sum the given geometric power series, finding an equivalent rational expression.
2. Take the derivative of the rational expression you obtained in #1.
3. Also take the derivative of the given power series.
4. Set the result of #1 equal to the result of #3.

27. \( F(x) = \sum_{k=0}^{\infty} \frac{x^{4k}}{2^{2k}} \)

28. \( G(x) = \sum_{k=0}^{\infty} \frac{5^{2k} x^{3k}}{\pi^{4k}} \)

29. \( H(x) = \sum_{k=0}^{\infty} \frac{(-\pi x)^k}{e^{2k}} \)
For problems #30 – 31, do each of the following:

5. Sum the given geometric power series, finding an equivalent rational expression.

6. Take the integral of the rational expression you obtained in #1.

7. Also take the integral of the given power series.

8. Set the result of #1 equal to the result of #3.

30. \( R(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^k}{2^{3k}} \)

31. \( G(x) = \sum_{k=0}^{\infty} \frac{5^{2k} x^{3k+2}}{\pi^{4k}} \)

For problems #32 – 34, find the first three terms of the given product (up to degree 2).

32. \( \left( \sum_{n=0}^{\infty} \frac{(n+1)x^n}{n!} \right) \left( \sum_{n=0}^{\infty} \frac{x^n}{(n+1)^3} \right) = ? \)

33. \( \left( \sum_{n=0}^{\infty} \frac{2^n x^n}{(n+1)!} \right) \left( \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{2^n(n+1)^2} \right) = ? \)

34. \( \left( \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \right) \left( \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \right) = ? \)