For problems # 1 – 6, find the best-fit quadratic function $T_2(x)$ to the given function at the given point.

1. $y = \cos x$ at $(0,1)$  
2. $y = e^x$ at $(0,1)$  
3. $g(x) = \cosh x$ at $x = 0$

4. $h(x) = \sqrt{1 + x^2}$ at $x = 0$  
5. $F(x) = \frac{1}{x^2 - 2x + 2}$ at $(1,1)$  
6. $R(x) = \frac{-2}{6x^2 - 12x + 7}$ at $(1,-2)$

For problems # 7 – 10, find the best-fit cubic function $T_3(x)$ to the given function at the given point.

7. $T(x) = \tan x$ at $x = 0$  
8. $S(x) = \sinh x$ at $x = 0$  
9. $L(x) = \ln x$ at $x = e^2$

10. $c(x) = \cot x$ at $x = \frac{\pi}{2}$  
11. $r(x) = \frac{2x^2}{x^4 + 2}$ at $x = 1$.

*12. Find the Taylor polynomial of degree 4 that approximates $\psi(x) = \ln(2 + x^2)$ near $x = 0$.

13. Find $T_{14}(x)$, the 14th degree Taylor polynomial approximation to $r(x) = \frac{2x^2}{x^4 + 2}$ at $x = 0$.

14. Let $r(x) = \frac{2x^2}{x^4 + 2}$, as in our last problem. Use the Lagrange remainder to find a bound on the error in approximating $r\left(\frac{1}{4}\right)$ by $T_{10}\left(\frac{1}{4}\right)$. Assume that $|r^{(14)}(x)| \leq 10^{11}$ for all $x$ in the interval $\left[0, \frac{1}{4}\right]$.

*(You calculated this Taylor polynomial in problem #13).*

15. Find $T_{31}(x)$, the 31st degree Taylor polynomial approximation to $Q(x) = \frac{x^3}{1 - x^7}$ at $x = 0$. 

16. Let \( Q(x) = \frac{x^3}{1-x^7} \), as in our last problem. Use the Lagrange remainder to find a bound on the error in approximating \( Q\left(\frac{1}{3}\right) \) by \( T_{24}\left(\frac{1}{3}\right) \). Assume that \( |Q^{(3)}(x)| \leq 3^9 \) for all \( x \) in the interval \( \left[0, \frac{1}{3}\right] \).

(You calculated this Taylor polynomial in problem #15).

17. Find \( T_{13}(x) \), the 13th degree Taylor polynomial approximation to \( R(x) = \frac{4x}{x^3 + 4} \) at \( x = 0 \).

18. Let \( R(x) = \frac{4x}{x^3 + 4} \), as in our last problem. Use the Lagrange remainder to find a bound on the error in approximating \( r\left(\frac{1}{3}\right) \) by \( T_{10}\left(\frac{1}{3}\right) \). Assume that \( |r^{(13)}(x)| \leq 3^{19} \) for all \( x \) in the interval \( \left[0, \frac{1}{3}\right] \).

(You calculated this Taylor polynomial in problem #17).

For problems # 19 – 25, find the Maclaurin series expansion of the given function and find its interval of convergence.

19. \( A(x) = \tan^{-1} x \)
20. \( f(x) = \frac{\tan^{-1} x}{x} \) \hspace{1cm} Hint: Use the results of #19.

21. \( S(x) = \sinh x \)
22. \( g(x) = \frac{\sinh x - x}{x^3} \)
23. \( g(x) = \frac{x - \sin x}{x^3} \)

24. \( C(x) = \cosh x \)
25. \( g(x) = \frac{\cosh x - 1}{x} \).

26. Use the results of #23 to calculate \( \lim_{x \to 0} \frac{x - \sin x}{x^3} \).

27. Use the results of #25 to calculate \( \lim_{x \to 0} \frac{\cosh x - 1}{x} \).
28. Find the Taylor series expansion of \( h(x) = (3x+1)^2 \), centered at \( x = 1 \).

29. Find the Maclaurin series expansion of \( F(x) = x^2 e^{-x^3} \), and its interval of convergence.

30. Find the Maclaurin series expansion of \( G(x) = x^3 \cos(2x^2) \), and its interval of convergence.

31. Find the Maclaurin series expansion of \( f(x) = \frac{7x+3}{x^2-1} \). (Hint – partial fractions!)

32. Find the Maclaurin series expansion of \( g(x) = \frac{x+23}{x^2-3x-10} \). (Hint – partial fractions!)

33. Find the Maclaurin series expansion of \( Q(x) = \frac{2x+36}{x^2+x-12} \). (Hint – partial fractions!)

34. Find the first three nonzero terms of the Maclaurin series expansion of \( H(x) = e^x \cos x \)
   
   by multiplying the series we’ve previously found for \( e^x \) and for \( \sin x \).

35. Find the first four nonzero terms of the Maclaurin series expansion of \( \zeta(x) = \frac{e^{-x}}{1+x^2} \),
   
   by multiplying the series for \( e^{-x} \) and the (geometric) series for \( \frac{1}{1+x^2} \).