2.2 Separable Variables

Note: Review your integration methods, especially integration by parts and partial fraction decomposition.

The simplest type of DE to solve is \( \frac{dy}{dx} = g(x) \).

*Example: Solve \( \frac{dy}{dx} = (x + 1)^2 \)

This is a special case of a DE which is **separable**. A first-order DE of the form \( \frac{dy}{dx} = g(x)h(y) \) is said to be separable; i.e. \( \frac{dy}{dx} = f(x, y) \) where \( f(x,y) \) is a product of a function of \( x \) and a function of \( y \).

*Example: Solve \( \frac{dy}{dx} + 2xy = 0 \). Write the solution in explicit form.

*Example: Solve \( \sin 3x \frac{dx}{dy} + 2y \cos^3 3x = 0 \). Write the solution in explicit form.
*Example: Solve $e^x y \frac{dy}{dx} = e^{-y} + e^{-2x-y}$.

Write the solution in implicit form.
*Example: Solve the IVP. \( \frac{dy}{dx} = \frac{y^2 - 1}{x^3 - 1} \), \( y(2) = 2 \). Write the solution in explicit form.

\[ \frac{dy}{dx} = \frac{y^2 - 1}{x^3 - 1} \]

\( y(2) = 2 \)

\[ \text{Write the solution in explicit form.} \]

\[ \text{Beware of losing a solution.} \]

Notice that in the previous example, \( y = 1 \) and \( y = -1 \) are constant solutions. We would get \( y = 1 \) from the family of solutions if \( c = \underline{} \), but we would not get \( y = -1 \) from the family.

\( y = -1 \) is a _________ solution.)

So determine any constant solutions before you rearrange the DE. They will be the zeros of \( \frac{dy}{dx} = f(x, y) \).