Instructions:

- Please turn off and put away all cell phones and any listening devices.
- Each question is weighted the same, and graded using a 5 point rubric. The exam is worth 100 points.
- Round all probabilities to three significant digits unless otherwise stated.
- Work should be clearly labeled and easy to follow.
- Partial credit will only be given for work showing significant progress toward the correct answer.
- You may use a graphing calculator on this exam.
- Work in pen will not be accepted.
- You may use one 8.5" x 11", or smaller handwritten sheet of notes. Both sides may be used, but photocopies of any kind are not allowed.
- Do not hesitate to ask the instructor if you do not understand what a question is asking, or if you have shown enough work.

Date: __________  Name: __________

Class(circle one):  4:20  6:00
Round all probabilities to three significant digits unless otherwise stated.

1. The academic majors for college students in the U.S. are distributed as follows: 45% humanities, 25% information technology, 20% psychology/sociology, 8% science, 2% math.
   
   \[ P(\text{not psych}) = 1 - .2 = .8 \]

   If you pick 4 college students at random, what is the probability that:

   a. The first student is the only one majoring in psychology/sociology.

   \[ P(\text{exactly one psych/soc major}) = \binom{4}{1}(.2)(.8)^3 = \frac{6.102}{1024} = .006 \]

   b. At least one is majoring in science.

   \[ P(\text{at least 1 sci}) = 1 - P(\text{none are sci}) = 1 - (.92)^4 = .284 \]

2. The results shown in the table below are from clinical trials of a blood test for pregnancy.

<table>
<thead>
<tr>
<th>Pregnancy Test Results</th>
<th>Subject is actually pregnant</th>
<th>Subject is not actually pregnant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive Test Results</td>
<td>80</td>
<td>3</td>
</tr>
<tr>
<td>test indicates the subject is pregnant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Negative Test Results</td>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>test indicates the subject is not pregnant</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   a. If 3 subjects are randomly selected with replacement, find the probability that they all have positive test results.

   \[ P(\text{pos. test}) = \frac{\frac{8}{9}}{99} \cdot \frac{8}{9} \cdot \frac{8}{9} = \frac{589}{99} \]

   (calc .589 2894871)

   b. If 4 subjects are randomly selected without replacement, find the probability that all of them are not actually pregnant.

   \[ P(\text{not pregnant}) = \frac{14}{99} \cdot \frac{13}{98} \cdot \frac{12}{97} \cdot \frac{11}{96} = \frac{.000266}{10^{-4}} \]

   (calc 2.659139257 x10^{-4})
3. For the following problems below, use a permutation or a combination. Explain why you choose that counting technique, and then do the problem.

a. How many different ways are there to select a jury of 12 people from a pool of 20?

\[
\begin{align*}
\text{Order does not } & \rightarrow \text{ Combination} \\
\text{mater} & \\
20C_{12} = \frac{20!}{(20-12)!12!} = \frac{20!}{8!12!} \\
& = \frac{12,597,000!}{12!} \\
& = 1,259,700 \text{ ways}
\end{align*}
\]

b. A disc jockey has 15 songs to choose from and can only play 8 of them in the next hour. How many different playlists are possible?

\[
\begin{align*}
\text{Order matters } & \rightarrow \text{ Permutation} \\
15P_8 = \frac{15!}{(15-8)!} = \frac{15!}{7!} \\
& = 2,559,459,200 \text{ different playlists}
\end{align*}
\]

4. The table below has the results of a study in which researchers examined a child's IQ and the presence of a specific gene in the child for a SRS of 102 children.

<table>
<thead>
<tr>
<th>Gene Present</th>
<th>Gene Not Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>High IQ</td>
<td>33</td>
</tr>
<tr>
<td>Normal IQ</td>
<td>39</td>
</tr>
</tbody>
</table>

\[
72 \quad 30 \quad 102
\]

One of the subjects in the study was randomly selected.

a. Find \( P(\text{gene is present}) = \frac{72}{102} = 0.706 \) \( (\text{calc.} \ 0.7058823529) \)

b. Find \( P(\text{gene is not present} \mid \text{high IQ}) = \frac{19}{52} = 0.365 \) \( (\text{calc.} \ 0.3653846154) \)
5. The access code for a car's security system consists of four digits. Each digit can be any number from 0 to 9. How many access codes are possible if

a. each digit can be used only once and not repeated?

\[
\begin{align*}
10 & \cdot 9 & \cdot 8 & \cdot 7 \\
\text{possible codes} & = & 5040
\end{align*}
\]

\[
P_Y = \frac{12}{(10-4)!} = \frac{1}{6!} = 5040
\]

b. Each digit can be repeated?

\[
10 \times 10 \times 10 \times 10 = 10^4 = 10,000 \text{ possible codes}
\]

c. each digit can be repeated, but the 1st digit cannot be 0 or 1?

\[
8 \times 10 \times 10 \times 10 = 8,000 \text{ possible codes}
\]

6. The following table summarizes blood groups and Rh types for 54 subjects.

<table>
<thead>
<tr>
<th>Type</th>
<th>A</th>
<th>B</th>
<th>AB</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rh^+</td>
<td>22</td>
<td>18</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Rh^-</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{Total} & = 25 + 20 + 6 + 3 + 54 \\
\text{Group A} & = 46 \\
\text{Group B} & = 20
\end{align*}
\]

a. If one subject is chosen at random, what is the probability the subject is group A or is group B?

\[
P(\text{group A or group B}) = \frac{25 + 20}{54} = \frac{45}{54} = \frac{5}{6} = 0.833
\]

\[
\text{Calc. } 0.833
\]

b. If one subject is chosen at random, what is the probability the subject is Group AB and Rh^+?

\[
P(\text{group AB and Rh}^+) = \frac{4}{54} = \frac{1}{13.5} = 0.0741
\]

\[
\text{Calc. } 0.074074
\]
7. The probability that a particular knee surgery is successful is 0.775. Assuming these events are independent, if six patients get this surgery, what are the chances the following occurs:

a. all six of the knee surgeries are successful.

\[ P(\text{all success}) = (0.775)^6 = 0.217 \]

(calc 0.216757034)

b. at least one of the six surgeries is unsuccessful.

\[ P(\text{at least 1 unsuccessful}) = 1 - P(\text{none unsuccessful}) = 1 - P(\text{all successful}) = 1 - (0.775)^6 = 0.783 \]

(Or 1 - 0.217 from part a)

8. A survey asked 2850 people whether they were involved in any type of charity work. The results are listed in the table below.

<table>
<thead>
<tr>
<th>Involved in Charity Work?</th>
<th>Frequently</th>
<th>Occasionally</th>
<th>Not at All</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Males</td>
<td>221</td>
<td>456</td>
<td>795</td>
<td>1472</td>
</tr>
<tr>
<td>Females</td>
<td>207</td>
<td>430</td>
<td>741</td>
<td>1378</td>
</tr>
<tr>
<td>Total</td>
<td>428</td>
<td>886</td>
<td>1536</td>
<td>2850</td>
</tr>
</tbody>
</table>

a. Fill in the missing entries in the table above.

Suppose an subject is selected at random. Find each of the following probabilities.

b. The probability that the subject is a female or not at all involved in charity work.

\[ P(\text{female or Not at All}) = \frac{1378 + 1536 - 741}{2850} = \frac{2173}{2850} = 0.762 \]

(calc 0.762450409)

c. The probability that the subject is a male given that they are frequently involved.

\[ P(\text{male | frequently}) = \frac{221}{428} = 0.516 \]

(calc 0.5163551402)
9. In a survey of consumers aged 12 or older, respondents were asked how many cell phones were in use by the household. Among the respondents, 14 answered "none," 198 said "one," 502 said "two," 151 said "three," and 85 responded with "four or more."

\[ P(\text{more than 2}) = \frac{161 + 85}{950} = \frac{236}{950} = 0.248 \]

\[ \text{(Calc: 0.2484210526)} \]

b. Find the probability that a randomly selected household has exactly 1 cell phone in use.

\[ P(\text{exactly 1}) = \frac{198}{950} = 0.208 \]

\[ \text{(Calc: 0.2084210526)} \]

10. Texas Instruments produced a batch of 10,000 TI-84 Plus calculators that included exactly 200 that were defective. A simple random sample of one hundred of them are selected for shipping.

a. Explain why or why not the 5% Guideline for Cumbersome Calculations can be used in this problem.

\[ \frac{100}{10,000} = 0.01 = 1\% < 5\% , \text{ so the 5\% Guideline can be used to treat events as independent.} \]

b. Assuming 100 of the calculators are selected for shipping, what is the probability they are all good?

\[ P(\text{all good}) = \left(\frac{9,800}{10,000}\right)^{100} = 0.133 \]

\[ \text{(Calc: 0.1326195559)} \]