Math B22
Spring 2014

Exam 3

"I forgot to make a back-up copy of my brain, so everything I learned last semester was lost."

Instructions:

• Please turn off and put away all cell phones and any listening devices.
• Each question is weighted the same, and graded using a 5 point rubric. The exam is worth 200 points.
• Work should be clearly labeled and easy to follow.
• Partial credit will only be given for work showing significant progress toward the correct answer.
• You may use a graphing calculator on this exam.
• Work in pen will not be accepted.
• You may use one 8.5" x 11", or smaller handwritten sheet of notes. Both sides may be used, but photocopies of any kind are not allowed.
• Do not hesitate to ask the instructor if you do not understand what a question is asking.

Round all probabilities to three significant digits

Date: __________  Name: __________

Class(circle one):  4:20  6:00
1. The average person uses 123 gallons of water daily. The standard deviation is 21 gallons and the amount of water used is normally distributed. Find the probability that 15 randomly selected people have a mean water usage between 120 and 126 gallons.

\[ \mu = 123 \text{ gal} \]
\[ \sigma = \frac{21}{\sqrt{15}} \text{ gal} \]

\[ P(120 \leq \bar{X} \leq 126) = \frac{120 - 123}{\frac{21}{\sqrt{15}}} \]

\[ \text{normalcdf}(120, 126, 123, \frac{21}{\sqrt{15}}) \]

(Calc 0.4199304569)

2. There are 400 typographical errors randomly distributed in a 500 page manuscript.

   a. Find the mean number of typos per page.

   \[ \mu = \frac{400}{500} \text{ typos per page} = \frac{4}{5} = 0.8 \]

   b. Find the probability that on any given page there are exactly 3 typos. Is it unlikely to get exactly 3 typos? Explain.

   \[ P(X = 3) = 0.0383 \]

   Unlikely since \( 0.0383 < 0.05 \).

   (Calc 0.0383427383)

   (Poisson pdf (.8, 3))

   c. Find the probability that on any given page there are more than 3 typos

   \[ P(X > 3) = P(X \geq 4) = 1 - P(X \leq 3) = 0.00908 \]

   \( 1 - \text{poisson cdf (.8, 3)} \)

   (Calc 0.0090798578)
3. The mean lifetime of a wristwatch is 25 months with a standard deviation of 5 months. If the distribution is normal, for how many months should a guarantee be made if the manufacturer does not want to exchange more than 8% of the watches?

\[ \begin{align*}
\mu &= 25 \text{ mos} \\
\sigma &= 5 \text{ mos}
\end{align*} \]

\[ X = \text{inv Norm (.08, 25, 5)} \]

\[ X = 17.97 \rightarrow \text{Guarantee should be for 18 months} \]

4. There is a 0.99925 probability that a randomly selected 35-year-old female lives through the year. A life insurance company charges $360 annually for a policy that pays out $100,000 as a death benefit.

a. Let \( x \) = "winnings" and construct a probability distribution.

\[ \begin{array}{c|c|c}
\text{event} & P(x) & \text{probability} \\
\hline
\text{lives} & $-360 & 0.99925 \\
\text{dies} & $99,640 & 0.00075
\end{array} \]

\[ P(\text{lives}) = 0.99925 \]
\[ P(\text{dies}) = 1 - 0.99925 = 0.00075 \]

\[ \text{Net gain} = 100,000 - 360 = 99,640 \]

b. Determine the expected value from the perspective of a 35-year-old female.

\[ E = -360(0.99925) + 99,640(0.00075) = \$ -285 \]

\[ \text{Or} \rightarrow E = \mu = \$ -285 \]

\[ \text{uno calc} \]
\[ 1 \text{-var stats x, x} \]
5. A researcher forms random groups with five males in each group. The random variable $x$ is the number of males in the group who have color blindness in the probability distribution given below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$P(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.659</td>
</tr>
<tr>
<td>1</td>
<td>0.287</td>
</tr>
<tr>
<td>2</td>
<td>0.050</td>
</tr>
<tr>
<td>3</td>
<td>0.004</td>
</tr>
<tr>
<td>4</td>
<td>0.001</td>
</tr>
<tr>
<td>5</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mean</th>
<th>4 males</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>0.6 males</td>
</tr>
</tbody>
</table>

a. Find the mean and the standard deviation.

b. Find the probability of getting 1 or 2 colorblind males among groups of 5 males.

\[
P(1 \text{ or } 2) = P(x=1) + P(x=2) = 0.287 + 0.050 = 0.337
\]

c. Find the probability needed to see if 3 is an unusually high number of colorblind males. Is 3 unusually high? Explain.

\[
P(x \geq 3) = P(x=3) + P(x=4) + P(x=5) = 0.004 + 0.001 = 0.005
\]

Yes, unusually high since 0.005 < 0.05

6. The average teacher's salary in North Dakota is $35,441. Assume the salaries are normally distributed with a standard deviation of $5100.

a. What is the probability that a randomly selected teacher has a salary greater than $45,000?

\[
P(\bar{x} > 45,000) = \Phi \left( \frac{45,000 - 35,441}{5100} \right) = \Phi(1.88)
\]

(calculate 0.0304435339)

b. You randomly select 75 teachers. What is the probability that their mean salary is less than $35,000?

\[
P(\bar{x} < 35,000) = \Phi \left( \frac{35,000 - 35,441}{\frac{5100}{\sqrt{75}}} \right) = \Phi(-0.94)
\]

(calculate 0.2269715483)
7. The mean number of miles driven per vehicle annually in the U.S. is 12,494. The annual mileage is normally distributed with a standard deviation of 1290 miles. What percentage of vehicles are driven more than 14,000 miles?

\[ P(X > 14,000) = 1 - 0.122 = 12.2\% \]

\[ \text{normalcdf}(14,000, 10^{99}, 12,494, 1290) \]

8. Eighty-five percent of Americans favor spending government money to develop alternative sources of fuel for automobiles. Suppose 120 Americans are randomly selected.

a. Find the probability of getting fewer than 90 Americans that favor spending government money.

\[ n = 120 \]
\[ p = 0.85 \]
\[ P(X < 90) = P(X \leq 89) = 0.00141 \]

\[ \text{binomcdf}(120, 0.85, 89) \]

\[ (\text{calc} \ 0.0014094922) \]

b. Find the probability that at least 100 favor spending government money.

\[ P(X \geq 100) = 1 - P(X \leq 99) = 0.744 \]

\[ 1 - \text{binomcdf}(120, 0.85, 99) \]

\[ (\text{calc} \ 0.7443171761) \]
9. Based on Mendel's genetic experiments, the probability of a pea having a green pod is 0.75. Suppose a group of 240 offspring peas are generated.

\[ n = 240, \ p = 0.75, \ \bar{q} = 0.25 \]

a. Find the mean and standard deviation for the numbers of green pods in a group of 240 offspring peas.

\[ \mu = n \bar{p} = 240 \times 0.75 = 180 \text{ green pods} \]
\[ \sigma = \sqrt{n \bar{p} \bar{q}} = \sqrt{240 \times 0.75 \times 0.25} = 6.7 \text{ green pods} \]

b. Use the range rule of thumb to determine if it is unusual to get 200 green pods from a group of 240 offspring peas.

\[ \text{max} = \mu + 2\sigma = 180 + 2(6.7) = 193.4 \]
\[ \text{min} = \mu - 2\sigma = 180 - 2(6.7) = 166.6 \]

\[ 166.6 \leq x \leq 193.4 \]

Since 200 is outside the range of usual \( x \)-values, getting 200 green pods is unusual.

10. The average waiting time to be seated at a popular restaurant is 23.5 minutes with a standard deviation of 3.6 minutes. Assume the wait times are normally distributed. Find the probability a customer will have to wait the following times.

a. Between 15 and 22 minutes.

\[ \mu = 23.5 \text{ min} \]
\[ \sigma = 3.6 \text{ min} \]
\[ P(15 \leq x \leq 22) = 0.329 \]

\[ \text{normcdf}(15, 22, 23.5, 3.6) \]
\[ (\text{calc. } 0.329351037) \]

b. Less than 18 minutes.

\[ \mu = 23.5 \text{ min} \]
\[ \sigma = 3.6 \text{ min} \]
\[ P(x < 18) = 0.0633 \]

\[ \text{normcdf}(-10.99, 18, 23.5, 3.6) \]
\[ (\text{calc. } 0.0632838702) \]