Practice Exam 2: Chapter 4

The following questions are intended to help you prepare for the exam, however they do not cover all possible topics for the exam. You should review your notes, textbook and homework problems.

1. The following is a contingency table with the counts (in thousands) of earned degrees in the United States in a recent year, classified by level and by the gender of the degree recipient:

<table>
<thead>
<tr>
<th></th>
<th>Bachelor's</th>
<th>Master's</th>
<th>Professional</th>
<th>Doctorate</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>616</td>
<td>194</td>
<td>30</td>
<td>16</td>
<td>856</td>
</tr>
<tr>
<td>Male</td>
<td>529</td>
<td>171</td>
<td>44</td>
<td>26</td>
<td>770</td>
</tr>
<tr>
<td>Total</td>
<td>1145</td>
<td>365</td>
<td>74</td>
<td>42</td>
<td>1626</td>
</tr>
</tbody>
</table>

a) Fill in the missing entries

b) If you chose a degree recipient at random, what is the probability that the person you choose is a male?

\[ P(\text{male}) = \frac{770}{1626} = 0.474 \]

c) What is the probability that you chose a person who received a master’s degree?

\[ P(\text{master's degree}) = \frac{365}{1626} = 0.224 \]

d) What is the probability that a person chosen at random is a male and earned a master’s degree?

\[ P(\text{male and master's degree}) = \frac{171}{1626} = 0.105 \]

e) What is the probability that a person chosen at random is either male or earned a master’s degree?

\[ P(\text{male or earned master's}) = \frac{770 + 365 - 171}{1626} = \frac{964}{1626} = 0.593 \]

f) What is the probability that a person earned a master’s degree given that the person is a male?

\[ P(\text{master's | male}) = \frac{171}{770} = 0.222 \]

g) What is the probability that a person is a male given that the person earned a master’s degree?

\[ P(\text{male | masters}) = \frac{171}{365} = 0.468 \]

2. Events A and B are defined on a common sample space. If \( P(A) = 0.7 \), and \( P(B) = 0.6 \), and A and B are independent, then find \( P(A \text{ and } B) \).

\[ P(A \text{ and } B) = P(A) \cdot P(B) = 0.7 \cdot 0.6 = 0.42 \]

3. Events A and B are defined on a common sample space. If \( P(A) = 0.4 \), and \( P(B) = 0.3 \), and A and B are disjoint, then find \( P(A \text{ or } B) \).

\[ P(A \text{ or } B) = P(A) + P(B) = 0.4 + 0.3 = 0.7 \]
4. The table below classifies institutions of higher education in the United States by region and type:

<table>
<thead>
<tr>
<th></th>
<th>Public</th>
<th>Private</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northeast</td>
<td>224</td>
<td>533</td>
<td>757</td>
</tr>
<tr>
<td>Midwest</td>
<td>366</td>
<td>427</td>
<td>793</td>
</tr>
<tr>
<td>South</td>
<td>500</td>
<td>520</td>
<td>1020</td>
</tr>
<tr>
<td>West</td>
<td>410</td>
<td>270</td>
<td>680</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1500</strong></td>
<td><strong>1750</strong></td>
<td><strong>3250</strong></td>
</tr>
</tbody>
</table>

Suppose an institution of higher education is selected at random. Determine the probability that the institution is

a) in the West \( P(\text{west}) = \frac{680}{3250} = 0.209 \)

b) a private school, given that it is in the West \( P(\text{private | west}) = \frac{270}{680} = 0.397 \)

c) in the West and is a private school \( P(\text{west and private}) = \frac{270}{3250} = 0.083 \)

d) a private school \( P(\text{private}) = \frac{1750}{3250} = 0.538 \)

e) in the West, given that it is a private school \( P(\text{west | private}) = \frac{270}{1750} = 0.154 \)

5. In a recent study, these data were obtained in response to the question, “Do you favor the proposal of the school’s combining the elementary and middle school students in one building?”

<table>
<thead>
<tr>
<th></th>
<th>Yes</th>
<th>No</th>
<th>No opinion</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Males</td>
<td>72</td>
<td>81</td>
<td>5</td>
<td>158</td>
</tr>
<tr>
<td>Females</td>
<td>103</td>
<td>68</td>
<td>7</td>
<td>178</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>175</strong></td>
<td><strong>149</strong></td>
<td><strong>12</strong></td>
<td><strong>336</strong></td>
</tr>
</tbody>
</table>

If a person is selected at random, find these probabilities.

a. The person has no opinion. \( P(\text{no opinion}) = \frac{12}{336} = 0.0357 \)

b. The person is a male or is against the issue. \( P(\text{male or no}) = \frac{158 + 149 - 81}{336} = \frac{226}{336} = 0.673 \)

c. The person is in favor of the issue, given that a female was selected. \( P(\text{yes | female}) = \frac{103}{176} = 0.579 \)

d. The person is a female and has no opinion. \( P(\text{female and no opinion}) = \frac{7}{336} = 0.0208 \)
6. In a scientific study there are eight guinea pigs, five of which are pregnant. If three are selected at random without replacement, find the probability that all are pregnant.

\[ n = 8 \]
\[ \text{select 3} \]
\[ P(\text{all pregnant}) = \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} = \frac{60}{336} = 0.179 \]

7. According to several sources, the probability that San Francisco has a major earthquake (magnitude 7.5 or larger) in any one year is 2%. Assuming these events are independent, find the probability that San Francisco has at least one major earthquake in the next 25 years.

\[ P(\text{quake}) = 0.02 \]
\[ P(\text{no quake}) = 0.98 \]
\[ n = 25 \]
\[ P(\text{at least 1}) = 1 - P(\text{no quakes}) = 1 - 0.98^{25} = 0.397 \]

8. An investment corporation is formed with enough capital to finance 9 independent ventures. If the chance of being successful on any one venture is 25%, what are the corporation's chances of having:

- The first one is the only success?
  \[ P(\text{1st only}) = P(\text{success}) = 0.25 \cdot (0.75)^8 = 0.025 \]

- At least one successful venture?
  \[ P(\text{at least 1 success}) = 1 - P(\text{no success}) = 1 - P(\text{all fail}) = 1 - (0.75)^9 = 0.925 \]

- All nine ventures unsuccessful?
  \[ P(\text{all fail}) = (0.75)^9 = 0.075 \]

9. The Masterfoods company says that before the introduction of purple, yellow candies made up 20% of their plain M&M's, red another 20%, and orange, blue, and green each made up 10%. The rest were brown. If you pick three M&M's at random, what is the probability that:

- They are all brown?
  \[ P(\text{all brown}) = \frac{3 \cdot 2}{13} \cdot \frac{2}{12} = 0.027 \]

- At least one is green?
  \[ P(\text{at least 1 green}) = 1 - P(\text{no green}) = 1 - \frac{9 \cdot 8 \cdot 7}{13 \cdot 12 \cdot 11} = 0.271 \]

- The third one is the first one that's red?
  \[ P(\text{no red, no red, red}) = \frac{8 \cdot 7}{13 \cdot 12} = 0.128 \]

- None are yellow?
  \[ P(\text{no yellow}) = \frac{8 \cdot 7 \cdot 6}{13 \cdot 12 \cdot 11} = 0.512 \]

Total = 100%

10. Suppose there are four independent events with probabilities: event 1 = 0.20, event 2 = 0.15, event 3 = 0.50, event 4 = 0.90. What is the probability of at least one of these events occurring?

\[ P(\text{at least 1 occurs}) = 1 - P(\text{none of them occurs}) = 1 - 0.8 \cdot 0.85 \cdot 0.5 \cdot 0.10 = 0.906 \]
11. Evaluate the following:

a. \( 6! = 720 \)
   \[ \frac{8!}{(4!2!)} = 840 \]

b. \[ \binom{25}{7} = \frac{\binom{25}{7}}{\binom{18}{7}} = \frac{25!}{18!7!} \]

c. \[ \binom{25}{7} = \frac{25!}{(25-7)!7!} = 48,070 \]

d. \[ \binom{25}{7} = \frac{25!}{(25-7)!} = \frac{25!}{18!} \]

12. The Virginia Win for Life lottery game requires that you select the correct six numbers between 1 and 42. What's the probability of winning this lottery by purchasing one ticket?

\[ P(\text{win}) = \frac{1}{5,245,786} \]

13. Find the number of different trifecta bets in a race with 8 horses.

\[ 8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 336 \]

14. a. If 12 newborn babies are randomly selected, how many different gender sequences are possible?

\[ 2^{12} = 4096 \]

b. How many different ways can 6 girls and 6 boys be arranged in a sequence?

\[ \frac{12!}{(6!)^2} = 924 \]

c. What is the probability of getting 6 girls and 6 boys when 12 babies are being born?

\[ P(6 \text{ girls and } 6 \text{ boys}) = \frac{924}{4096} = 0.22 \]