CHAPTER 2 - SETS, WHOLE NUMBERS AND FUNCTIONS

Ongoing Assessment 2-1: Describing Sets

1. (a) Either a list or set-builder notation may be used: 
   \{m, a, b, c, d, e, i, c, s\} or \{x \mid x \text{ is a letter in the word mathematics}\}.

   (b) Too much to list; use set-builder notation:
   \{x \mid x \text{ is a state in the United States, but } x \text{ is not Alaska or Hawaii}\}.

   (c) \{21, 22, 23, 24, \ldots\} or \{x \mid x \text{ is a natural number and } x > 20\} or \{x \mid x \in \mathbb{N} \text{ and } x > 20\}.

   (d) [Alaska, California, Hawaii, Nevada, Oregon, Washington] or \{x \mid x \text{ is a state in the United States that borders the Pacific Ocean}\}.

2. (a) \(P = \{a, b, c, d\}\).

   (b) \(\{1, 2\} \subset \{1, 2, 3, 4\}\). The symbol \(\subset\) refers to a proper subset.

   (c) \(\{0, 1\} \nsubseteq \{1, 2, 3, 4\}\). The symbol \(\nsubseteq\) refers to a subset.

   (d) \(0 \notin \{\}\) or \(0 \notin \emptyset\).

   (e) \(\{\}\ \neq \{\} \text{ or } \{\}\ \neq \emptyset\).

3. (a) Yes. \(\{1, 2, 3, 4, 5\} \sim \{m, n, o, p, q\}\) because there exists a one-to-one correspondence.

   (b) No. \(\{m, n, o, p, q\} \neq \{f, u, n\}\) because there is no one-to-one correspondence.

   (c) Yes. \(\{a, b, c, d, e, f, \ldots, m\} \sim \{1, 2, 3, \ldots, 18\}\) because both sets have an infinite number of elements and thus exhibit one-to-one correspondence.

   (d) No. \(\{x \mid x \text{ is a letter in the word mathematics}\} \neq \{1, 2, 3, 4, \ldots, 11\}\); there are only eight unduplicated letters in the word mathematics.

   (e) No. \(\{0, 0\} \neq \{2\}\); there is no one-to-one correspondence.

4. (a) The first element of the first set can be paired with any of the five in the second set, leaving four possible pairings for the second element, three for the third, two for the fourth, and one for the fifth. \(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120\) possible one-to-one correspondences.

   (b) There are \(6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720\) one-to-one correspondences.

   (c) There are \(n \cdot (n - 1) \cdot (n - 2) \ldots 2 \cdot 1\) possible one-to-one correspondences.

5. (a) If \(x\) must correspond to 5, then \(y\) may correspond to any of the four remaining elements of \(\{1, 2, 3, 4, 5\}\). \(x\) may correspond to any of the three remaining, etc. Then \(1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 24\) one-to-one correspondences.

   (b) There would be \(1 \cdot 1 \cdot 3 \cdot 2 \cdot 1 = 6\) one-to-one correspondences.

   (c) The set \(\{x, y, z\}\) could correspond to the set \(\{1, 3, 5\}\) in \(3 \cdot 2 \cdot 1 = 6\) ways. The set \(\{u, v\}\) could correspond with the set \(\{2, 4\}\) in \(2 \cdot 1 = 2\) ways. There would then be \(6 \cdot 2 = 12\) one-to-one correspondences.

6. (i) \(A = C\). The order of the elements does not matter.

   (ii) \(E = H\); they are both the null set.

   (iii) \(I = J\). Both represent the numbers \(1, 3, 5, 7, \ldots\).

7. (a) This is an arithmetic sequence with \(a_1 = 101\), \(a_n = 1100\), and \(d = 1\). Thus \(1100 = 101 + (n - 1) \cdot 1\); solving, \(n = 1000\). The cardinal number of the set is therefore \(1000\).

   (b) This is an arithmetic sequence with \(a_1 = 1\), \(a_n = 1001\), and \(d = 2\). Thus \(1001 = 1 + (n - 1) \cdot 2\); solving, \(n = 501\). The cardinal number of the set is therefore \(501\).

   (c) This is a geometric sequence with \(a_1 = 1\), \(a_n = 1024\), and \(r = 2\). Thus \(1024 = 1 \cdot 2^{n-1} \Rightarrow 2^{10} = 2^{n-1} \Rightarrow n - 1 = 10 \Rightarrow n = 11\). The cardinal number of the set is therefore \(11\).

   (d) If \(k = 1, 2, 3, \ldots, 100\), the cardinal number of the set \(\{x \mid x = k^2, k = 1, 2, 3, \ldots, 100\}\) is \(100\), since there are \(100\) elements in the set.

   (e) The set \(\{1, 2\}, \{3, 4\}, \{5, 6\}\) has three elements. \(\{1, 2\}, \{3, 4\}\), and \{5, 6\} are elements of the major set, even though individually they are themselves sets, thus the cardinal number is \(3\).
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7. (f) The set \( \{i + j \mid i \in \{1, 2, 3\} \text{ and } j \in \{1, 2, 3\}\} = \{(1 + 1), (1 + 2), (1 + 3), (2 + 1), \ldots, (3 + 3)\} \) has only five distinguishable elements: 2, 3, 4, 5, 6. The cardinal number is therefore 5.

8. \( \bar{A} \) represents all elements in \( U \) that are not in \( A \), or the set of all students with at least one grade that is not an \( A \). In set-builder notation, \( \bar{A} = \{x \mid x \text{ is a college student who does not have a straight-A average}\} \).

9. (a) A proper subset must have at least one less element than the set, so the maximum \( n(B) = 7 \).
   (b) If \( C \) had only one element, \( B \), to be a proper subset, would have 0 elements. Thus \( B \) would be the empty set \( \emptyset \).

10. (a) The sets are equal, so \( n(D) = 5 \).
    (b) The only way for \( C \) and \( D \) to be subsets of each other is if \( C = D \).

11. (a) \( \emptyset \). There are no subsets of the empty set.
    (b) \( \emptyset \). \{1\} is a set, while 1 and 2 are elements of the set \{1, 2\}.
    (c) \( \emptyset \). The empty set has no elements.
    (d) \( \emptyset \). \{1, 2\} is a subset of \{1, 2\}, not an element.
    (e) \( \in \). \(1024 = 2^{10} \) and \( 10 \in N \).
    (f) \( \in \). \(3 \cdot 1001 - 1 = 3002 \) and \( 1001 \in \N \).

12. (a) \( \emptyset \). 0 is not a set so cannot be a subset of the empty set, which in any event has no subsets.
    (b) \( \subseteq \). \{1\} is actually a proper subset, \( \subset \), of \{1, 2\}.
    (c) \( \subseteq \). Any set is a subset of itself.
    (d) \( \subseteq \). \emptyset is a subset of itself.
    (e) \( \subseteq \). \{1024\} is an element, not a subset.
    (f) \( \subseteq \). 3002 is an element, not a subset.

13. (a) Yes. Any set is a subset of itself, so if \( A = B \), then \( A \subseteq A \).
    (b) No. \( A \) could equal \( B \), then \( A \) would be a subset but not a proper subset of \( B \).
    (c) Yes. Any proper subset is also a subset.
    (d) No. \( A \) could be any of the proper subsets of \( B \), thus not equal to \( B \).

14. (a) Let \( A = \{a, b, c, d\} \) and let \( B = \{a, b, c, d\} \). Then \( A \subseteq B \) and since \( n(A) = 4 \) and \( n(B) = 4 \), \( 2 < 4 \).
    (b) \( \{1, 2, 3\} \subseteq \{1, 2, 3, \ldots, 100\} \) and since \( n(\{1, 2, 3\}) = 3 \) and \( n(\{1, 2, 3, \ldots, 100\}) = 100 \), \( 3 < 100 \).
    (c) \( \{\} \subseteq \{1, 2, 3\} \) and since \( n(\{\}) = 0 \) and \( n(\{1, 2, 3\}) = 3 \), \( 0 < 3 \).

15. There are 7 senators from which to pick the first subcommittee member, leaving 6 from which to pick the second, and then 5 remaining from which to pick the final subcommittee member. If the order of the senators was important (e.g., if there were to be a president, vice-president, and secretary) there would be \( 7 \cdot 6 \cdot 5 = 210 \) different ways to pick three senators from a group of seven.

   In this case, though, order is not important (e.g., Abel, Brooke, and Cox would form the same subcommittee as Brooke, Cox, and Abel). There are \( 3 \cdot 2 \cdot 1 = 6 \) ways to arrange three subcommittee members, so \( \frac{210}{6} = 35 \) possible subcommittees.

   Another way to solve this problem would be to list all the possible subsets of the Senate committee having exactly three elements. There would be 35 of these subsets.

16. There are 9 ways to choose the first number, and 9 ways (since there can be no repetitions) to choose the second. Then \( 9 \cdot 9 = 81 \) numbers.

Ongoing Assessment 2-2
Other Set Operations and Their Properties

1. (a) Set-builder notation describes the elements of the set, rather than listing them; i.e., \( \{x \mid x \in \N \text{ and } 4 \leq x \leq 9\} \), where \( \N \) represents the set of natural numbers, allows the set to be built.
   (b) This set has only three elements; i.e., \( \{15, 30, 45\} \) thus the elements may easily be listed.

2. \( A = \{1, 3, 5, \ldots\}; B = \{2, 4, 6, \ldots\}; C = \{1, 3, 5, \ldots\} \)
   (a) \( A \cup C \). Every element of \( C \) is in \( A \) or \( C \).
   (b) \( A \cap C \). All elements of \( C \) are common to \( A \) and \( C \).
2. (c) \( N \). Every natural number is in either \( A \) or \( B \).

(d) \( \emptyset \). There are no natural numbers in both \( A \) and \( B \).

3. For example, let \( U = \{e, q, u, o, i, t, y\} \), \( A = \{i, i, t, e\} \), \( B = \{t, i, t, e\} \), and \( C = \{q, u, e\} \)

(a) Yes. \( A \cap B = \{i, t, e\} \); \( B \cap A = \{t, i, e\} \), so the sets are equal. More formally, \( A \cap B = \{x \mid x \in A \text{ and } x \in B\} = B \cap A \).

(b) Yes. \( A \cup B = \{l, i, t, e\} = B \cup A \), so the sets are equal. I.e., \( A \cup B = \{x \mid x \in A \text{ or } x \in B\} = B \cup A \).

(c) Yes. \( B \cup C = \{i, i, t, e, q, u\} \); \( A \cup (B \cup C) = \{i, i, t, e, q, u\} \). \( A \cup B = \{i, i, t, e\} \); \( (A \cup B) \cup C = \{i, i, t, e, q, u\} \), so the sets are equal.

(d) Yes. \( A \cup \emptyset = \{l, i, t, e\} = A \), so the sets are equal.

(e) Yes. \( A \cup A = \{l, i, t, e\} \); \( A \cup \emptyset = \{l, i, t, e\} \), so the sets are equal.

(f) No. \( A \cap A = \{l, i, t, e\} \); \( A \cap \emptyset = \emptyset \). The sets are not equal.

4. (a) True. Let \( A = \{1, 2\} \); \( A \cup \emptyset = \{1, 2\} \).

(b) False. Let \( A = \{1, 2\} \) and \( B = \{2, 3\} \); \( A \cap B = \{1\} \); \( B \cap A = \{2\} \).

(c) True. Let \( A = \{1, 2\} \); \( A \cup A = \{1, 2\} \).

(d) False. Let \( U = \{1, 2\} \); \( A = \{1\} \); \( B = \{2\} \).

(e) True. Let \( A = \{1, 2\} \); \( B = \{3, 4\} \); \( C = \{5, 6\} \).

(f) False. Let \( A = \{1, 2\} \); \( B = \{2, 3, 4\} \); \( A \cup B = \{1, 2, 3, 4\} \).

(g) False. Let \( A = \{1, 2\} \); \( B = \{2, 3\} \).

5. (a) If \( B \subseteq A \), all elements of \( B \) must also be elements of \( A \), but there may be elements of \( A \) that are not elements of \( B \), so \( A \cap B = B \).

(b) If \( B \subseteq A \), then \( n(A) \geq n(B) \), so \( A \cup B = A \).

(c) There are no elements in \( B \) that are not also in \( A \), so \( B - A = \emptyset \).

(d) If \( B \subseteq A \), then there are no elements in \( A \) that are also in \( B \), so \( B \cap A = \emptyset \).
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6. (f) \((A \cup B) \cap C\)

\[ A \subseteq B \text{ because all the elements of } A \text{ are contained in } B \text{ but } A - B = \emptyset \text{ because there are no elements in } A \text{ that are not in } B. \]

(g) \((A \cap B) \cup C\)

\[ A \cap B \text{ is the set of all elements in } A \text{ that are not in } B. \]

(b) \((\bar{A} \cap B) \cup C\)

7. (a) \(S \cup \bar{S} = \{ x \mid x \in S \text{ or } x \in \bar{S} \} = U.\)

(b) \(\emptyset \cup S = \{ x \mid x \in \emptyset \text{ or } x \in S \} = S.\) I.e., \(\emptyset\) has no elements to add to \(S.\)

(c) If \(U\) is the universe the complement of \(U\) can have no elements, thus \(\bar{U} = \emptyset.\)

(d) If the empty set has no elements its complement must have all elements, thus \(\bar{\emptyset} = U.\)

(e) There are no elements common to \(S\) and \(\bar{S}\), so \(S \cap \bar{S} = \emptyset.\)

(f) Since there are no elements in the empty set there are none common to it and \(S,\) so \(\emptyset \cap \bar{S} = \emptyset.\)

8. (a) If \(A \cap B = \emptyset\) then \(A\) and \(B\) are disjoint sets and any element in \(A\) is not in \(B,\) so \(A - B = \{ x \mid x \in A \text{ and } x \notin B \} = A.\)

(b) If \(B = U\) there are no elements in \(A\) which are not in \(B,\) so \(A - B = \emptyset.\)

(c) If \(A = B\) there are no elements in one set which are not also in the other, so \(A - B = \emptyset.\)

(d) If \(A \subseteq B\) then all elements of \(A\) must also be in \(B,\) so \(A - B = \{ x \mid x \in A \text{ and } x \notin B \} = \emptyset.\)

9. (a) (i) Let \(A = \{1, 2, 3, 4\}\) and \(B = \{1, 2, 3, 4, 5\}.\)

\[ A \subseteq B \text{ because all the elements of } A \text{ are contained in } B \text{ but } A - B = \emptyset \text{ because there are no elements in } A \text{ that are not in } B. \]

(ii) Let \(A = \{2, 4, 6, 8\}\) and \(B = \{1, 2, 3, 4, 5, 6, 7, 8\}.\) The same argument as in (i) applies.

(b) Yes. By definition, \(A - B\) is the set of all elements in \(A\) that are not in \(B.\) If \(A - B = \emptyset\) this means there are no elements in \(A\) that are not in \(B.\) Thus \(A \subseteq B.\)

More formally, suppose \(A \notin B.\) Then there must be an element in \(A\) that is not in \(B,\) which implies that it is in \(A - B.\) This implies that \(A - B\) is not empty; i.e., a contradiction. Thus \(A \subseteq B.\)

10. (a) \(B \cap \bar{A} \text{ or } B - A; \text{i.e., } \{ x \mid x \in B \text{ but } x \notin A \}.\)

(b) \(A \cap B \text{ or } A \cap \bar{B}; \text{i.e., } \{ x \mid x \notin A \text{ or } B \}.\)

(c) \((A \cap B) \cap \bar{C} \text{ or } (A \cap B) - C; \text{i.e., } \{ x \mid x \in A \text{ and } B \text{ but } x \notin C \}.\)

(d) \(A \cap C; \text{i.e., } \{ x \mid x \in A \text{ and } C \}.\)

(e) \((A \cup B) \cap C \text{ or } C - (A \cup B); \text{i.e., } \{ x \mid x \in A \text{ or } B, \text{ and } x \notin C \}.\)

(f) \([B \cup C] - A \text{ or } (A \cap B) \cap C \text{ or } [(B \cup C) - A] \cup (A \cap B) - C; \text{i.e., } \{ x \mid x \in B \text{ or } C \text{ but } x \notin A, \text{ or } x \in A \text{ and } B \text{ and } C \}.\)

11. (a) \(\bar{B}\) is the set of all elements in \(U\) that are not in \(B.\)

\[ A \cap B \text{ is the set of all elements common to } A \text{ and } B. \]

(b) \(A \cup B \text{ is the set of all elements in either } A \text{ or } B.\)

\[ A \cap B \text{ is the set of all other elements in } U. \]

(c) \(\bar{A}\) is the set of all elements in \(U\) that are not in \(A.

\[ A \cap B \text{ is the set of all elements common to both } A \text{ and } B. \text{ Their union, } (A \cap B) \cup \bar{A} \text{ is the set of all elements in either of these two.} \]
11. (c) 

(d) $A - B$ is the set of all elements in $A$ that are not in $B$.

12. (a) False:

(b) False:

(c) False:

13. (a) $A \cap B \cap C \subseteq A \cap B$ because all elements of $A \cap B \cap C$ are included in $A \cap B$.

(b) $A \cup B \subseteq A \cup B \cup C$ because all elements of $A \cup B$ are included in $A \cup B \cup C$.

(c) $(A \cup B) \cap C \subseteq A \cup B$ because all elements of $(A \cup B) \cap C$ are included in $A \cup B$.

(d) Neither, in general, is a subset of the other. There are no elements common to $A - B$ and $B - A$; thus one cannot be a subset of the other. If $A$ and $B$ are disjoint, then $A - B = A$ and $B - A = B$; these also are disjoint.

14. (a) (i) Greatest $n(A \cup B) - n(A) + n(B) = 5$ if $A$ and $B$ are disjoint.

(iii) Greatest $n(A \cap B) = n(B) = 2$ if $B \subseteq A$.

(iv) Greatest $n(A - B) = n(A) = 3$ if $A$ and $B$ are disjoint.

(b) (i) Greatest $n(A \cup B) = n + m$ if $A$ and $B$ are disjoint.

(ii) Greatest $n(A \cap B) = m$ if $B \subseteq A$, or $n$ if $A \subseteq B$.

(iii) Greatest $n(A - B) = m$ if $A$ and $B$ are disjoint.

(iv) Greatest $n(A - B) = n$ if $A$ and $B$ are disjoint.

15. (a) Use a Venn diagram:

(i) Enter 8 as $n(A \cap B)$;

(ii) $n(B) = 12$, but 8 of these are in $A \cap B$, so there are 4 elements in $B$ but not $A$;

(iii) $n(A \cup B) = 22$, but 12 are accounted for so there are 10 elements in $A$ but not in $B$; so

(iv) $n(A) = 10 + 8 = 18$.

(b) Use a Venn diagram:

(i) Enter 5 as $n(A \cap B)$;

(ii) $n(A) = 8$, but 5 of these are in $A \cap B$, so there are 3 elements in $A$ but not $B$;

(iii) $n(B) = 14$, but 5 of these are in $A \cap B$, so there are 9 elements in $B$ but not in $A$; so

(iv) $n(A \cup B) = 3 + 5 + 9 = 17$. 


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16. (a) (i) Greatest \( n(A \cup B \cup C) = n(A) + n(B) + n(C) = 4 + 5 + 6 = 15 \), if \( A, B, \) and \( C \) are disjoint.
(ii) Least \( n(A \cup B \cup C) = n(C) = 6 \), if \( A \subset B \subset C \).

(b) (i) Greatest \( n(A \cap B \cap C) = n(A) = 4 \), if \( A \subset B \subset C \).
(ii) Least \( n(A \cap B \cap C) = 0 \), if \( A, B, \) and \( C \) are disjoint.

17. (a) \( \overline{A \cup B} = \overline{A} \cap \overline{B} \)

(b) \( \overline{A \cap B} = A \cup B \)

(c) Let \( U = \{a, b, c, d\} \), \( A = \{a, b\} \), \( B = \{b, c\} \):
(i) \( \overline{A \cup B} = \{a, b, c\} \cap \{d\} \)
\( \overline{A \cap B} = \{c, d\} \cap \{a, d\} = \{d\} \).
(ii) \( \overline{A \cap B} = \{a, b, c\} \)
\( \overline{A \cup B} = \{c, d\} \cup \{a, d\} = \{a, c, d\} \).

18. Constructing a Venn diagram will help in visualization:

(a) \( B \cup S \) is the set of college basketball players more than 200 cm tall.

(b) \( \overline{S} \) is the set of humans who are not college students or who are college students less than or equal to 200 cm tall.

(c) \( B \cup S \) is the set of humans who are college basketball players or who are college students taller than 200 cm.

(d) \( B \cup S \) is the set of all humans who are not college basketball players and who are not college students taller than 200 cm.

(e) \( B \cap S \) is the set of all college students taller than 200 cm who are not basketball players.

(f) \( B \cap S \) is the set of all college basketball players less than or equal to 200 cm tall.

19. (a) The set of all Paxson 8th graders who are members of the band but not the choir, or \( B - C \).
(b) The set of all Paxson 8th graders who are members of both the band and the choir, or \( B \cap C \).
(c) The set of all Paxson 8th graders who are members of the choir but not the band, or \( C - B \).
(d) The set of all Paxson 8th graders who are neither members of the band nor of the choir, or \( B \cup C \).

20. Enter numbers in each region in the following order:

(i) \( n(A \cap B \cap C) = 3 \);
(ii) \( n(A \cap B) = 10 - 3 = 7 \);
(iii) \( n(A \cap C) = 8 - 3 = 5 \);
(iv) \( n(B \cap C) = 12 - 3 = 9 \);
(v) \( n(A) = 26 - 7 - 3 - 5 = 11 \);
(vi) \( n(B) = 32 - 7 - 3 - 9 = 13 \);
(vii) \( n(C) = 23 - 5 - 3 - 9 = 6 \); and
(viii) \( n(U) = 65 - 7 - 3 - 9 - 11 - 13 - 6 = 11 \).

21. Use a three-set Venn diagram, labeling the sets \( B \) (for basketball), \( V \) (volleyball), and \( S \) (soccer):

(i) Enter 2 in the region representing \( B \cap V \cap S \) (i.e., there were two who played all three sports);

(ii) Enter 1 in the region representing \( (B \cap V) - S \) (i.e., there was one who played basketball and volleyball but not soccer);

(iii) Enter 1 in the region representing \( (B \cap S) - V \) (i.e., there was one who played basketball and soccer but not volleyball);

(iv) Enter 2 in the region representing \( (V \cap S) - B \) (i.e., there were two who played volleyball and soccer but not basketball);
21. (v) Enter $7 - (1 + 1 + 2) = 3$ in the region representing $B - (V \cup S)$ (i.e., of the seven who played basketball, one also played volleyball, one also played soccer, and two also played both basketball and soccer -- leaving three who played basketball only);

(vi) Enter $9 - (1 + 2 + 2) = 4$ in the region representing $V - (B \cup S)$ (i.e., of the nine who played volleyball, one also played basketball, two also played soccer, and two also played both basketball and soccer -- leaving four who played volleyball only);

(vii) Enter $10 - (1 + 2 + 2) = 5$ in the region representing $S - (B \cup V)$ (i.e., of the ten who played soccer, one also played basketball, two also played volleyball, and two also played both basketball and volleyball -- leaving five who played soccer only).

There are then $3 + 4 + 5 + 1 + 2 + 2 = 18$ who played one or more sports.

22. In the Venn diagram below:

(i) There were 5 members who took both biology and mathematics;

(ii) Of the 18 who took mathematics 5 also took biology, leaving 13 who took mathematics only;

(iii) 8 took neither course, so of the total of 30 members there were $30 - (5 + 13 + 8) = 4$ who took biology but not mathematics.

23. In the Venn diagram below:

(i) "I" is the only letter contained in the set $A \cap B \cap C$ (i.e., the only letter common to Iowa, Hawaii, and Ohio);

(ii) "W" and "A" are the only letters contained in the set $(A \cap B) - C$ (i.e., the letters contained in both Iowa and Hawaii other than "T");

(iii) "O" is the only letter contained in the set $(A \cap C) - B$ (i.e., the letter contained in both Iowa and Ohio other than "T");

(iv) "H" is the only letter contained in the set $(B \cap C) - A$ (i.e., the letter contained in both Hawaii and Ohio other than "T");

(v) There are no letters contained in the sets $A - (B \cup C), B - (A \cup C), \text{ or } C - (A \cup B)$ (i.e., "I", "W", "A", "O", and "H" are the only letters used to form Iowa, Hawaii, and Ohio);

(vi) "T", "S", "N", and "G" are the letters in Washington not used in Iowa, Hawaii, or Ohio.

24. Answers may vary, but the following are possible:

(i) Elements in $A$ only, or $A - (B \cup C)$.

(ii) Elements in $B$ only, or $B - (A \cup C)$.

(iii) Elements in $C$ only, or $C - (A \cup B)$.

(iv) Elements in $A$ and $C$ but not $B$, or $(A \cap C) - B$.

(v) Elements in $B$ and $C$ but not $A$, or $(B \cap C) - A$.

(vi) Elements in $A$ and $B$ but not $C$, or $(A \cap B) - C$.

(vii) Elements in all three sets, or $A \cap B \cap C$.

(viii) Elements in neither $A$ nor $B$ nor $C$, or $A \cup B \cup C$.

25. (a) If all bikes needing new tires also need gear repairs, i.e., if $\{\text{T]RES\} \subset \{\text{GEARS\}}$, then $n(\{\text{TIRES\}) \cap \{\text{GEARS\})} = 20$ bikes.

(b) Adding the separate repairs gives $20 + 30 = 50$ bikes, so at least 10 were counted twice; i.e., 10 bikes needed both repairs.
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25. (c) If the maximum number of bikes needed both repairs then all 20 receiving tires were among the 30 having gear work, leaving 10 bikes that needed no service.

26. Generate the following Venn diagram in this order:

(i) The 50 who used all three cards;
(ii) The 60 who used only Gold and Thrift cards;
(iii) The 70 who used only Gold and Super cards;
(iv) The 240 - (80 + 50 + 70) = 40 who used only Gold cards;
(v) The 290 = (80 + 50 + 60) = 100 who used only Super cards; and
(vi) The 270 - (70 + 50 + 60) = 50 who used only Thrift cards.

The diagram indicates only 490 cardholders accounted for; so either there is some other type credit card used by the remaining ten people or the editor was right.

27. Generate the following Venn diagram in this order:

(i) The 4 who had A, B, and Rh antigens;
(ii) The 5 - 4 = 1 who had A and B antigens, but who was Rh negative;
(iii) The 31 - 4 = 27 who A antigens and was Rh positive;
(iv) The 11 - 4 = 7 who had B antigens and was Rh positive;
(v) The 40 - 27 - 4 - 1 = 8 who had A antigens only;
(vi) The 18 - 7 - 4 - 1 = 6 who had B antigens only; and
(vii) The 82 - 27 - 4 - 7 = 44 who were O positive.

\[ n(A \cup B \cup Rh) = 8 + 1 + 6 + 4 + 27 + 7 + 44 = 97. \] Thus the set of people who are O-negative is 100 - 97 - 3.

28. (a) False. Let \( A = \{a, b, c\}, B = \{1, 2, 3\} \). Then \( A \sim B \) but \( A \neq B \).

(b) False. Let \( A = B \). Then \( A \cup B = B \sim B \).

(c) False. Let \( A = \{1, 2, 3\}, B = \{1, 2, 3, 4\} \). Then \( A - B = \emptyset \) but \( A \neq B \).

(d) True. If \( B = A = \emptyset \), there are no elements in \( B \) which are not in \( A \). Thus \( B \subseteq A \).

(e) True. If \( A \subset B \), there is at least one element in \( B \) which is not in \( A \).

(f) False. Let \( A = \{1, 2, 3\}, B = \{a, b, c, d\} \). Then \( n(\{A\}) < n(\{B\}) \) but \( A \not\subset B \).

29. (a) \( I \cap W \) is the number of age 12 boys who want to play the infield = 10.

(b) \( T \cup E \) is the number of age 10 or 11 boys; \( C \) is the number of those who want to play catcher. \( C \cap (T \cup E) = 12 + 8 = 20 \)

(c) \( I \cup O \) is the number of boys who want to play the infield or the outfield; \( T \) is the number of age 10 boys. \( (I \cup O) \cap T = 28 + 14 = 42 \)

(d) \( T \cup E \) is the number of age 10 or 11 boys; \( O \) is the number of boys who want to play the outfield. \( (T \cup E) \cap O = 14 + 20 = 34 \).

30. The following Venn diagram helps in isolating the choices:

All picked the Cowboys to win their game, so their opponent cannot be among any of the other choices; the only team not picked was the Giants.

Phyllis and Paula both picked the Steelers, so their opponent cannot be among their other choices. This leaves the Jets.

Giants
Steelers
Cowboys
Jets
Packers

All

Winnipeg
30. Phyllis and Rashid both picked the Vikings which leaves the Packers as the only possible opponent.

Paula and Rashid both picked the Redskins which leaves the Bills as the only possible opponent.

Thus we have Cowboys vs Giants, Vikings vs Packers, Redskins vs Bills, and Jets vs Steelers.

31. If a Cartesian product is the set of all ordered pairs such that the first element of each pair is an element of the first set and the second element of each pair is an element of the second set:

(a) \( A \times B = \{ (x, a), (x, b), (x, c), (y, a), (y, b), (y, c) \} \).

(b) \( B \times A = \{ (a, x), (a, y), (b, x), (b, y), (c, x), (c, y) \} \).

(c) \( B \times \emptyset = \emptyset \). There are no elements in \( \emptyset \) with which to have a Cartesian product.

(d) \( (A \cup B) = \{ x, y, a, b, c \} \), thus \( (A \cup B) \times C = \{ (x, 0), (y, 0), (a, 0), (b, 0), (c, 0) \} \).

(e) \( B \times C = \{ (a, 0), (b, 0), (c, 0) \} \), thus \( A \cup (B \times C) = \{ x, y, a, b, c, 0 \} \).

32. (a) True. \( 1 + 1, 7 - 2 = (2, 5) \).

(b) True. The order of the same elements within a set is irrelevant.

(c) False. These are ordered pairs, thus order is relevant.

(d) False. The left side is an ordered pair, while the right side is a set.

33. (a) The first element of each ordered pair is \( a \), so \( C = \{ a \} \). The second element in the ordered pairs is, respectively, \( b, c, d \), and \( e \), so \( D = \{ b, c, d, e \} \).

(b) The first element in the first three ordered pairs is \( 1 \); in the second three is \( 2 \), so \( C = \{ 1, 2 \} \). The second element in the ordered pairs is, respectively, \( 1, 2, 3, 5 \), so \( D = \{ 1, 2, 3, 5 \} \)

(c) The numbers 0 and 1 appear in each ordered pair, so \( C \subseteq D = \{ 0, 1 \} \). (The order of the numbers in these sets is irrelevant.)

34. (a) Each of the five elements in \( A \) is paired with each of the four in \( B \) so there are \( 5 \times 4 = 20 \) elements.

(b) Each of the \( m \) elements in \( A \) are paired with each of the \( n \) in \( B \) so there are \( m \times n \) elements.

(c) \( A \times B \) has \( m \times n \) elements, each of which are paired with the \( p \) elements in \( C \) so there are \( m \times n \times p \) elements.

35. The number of games is equivalent to the number in the Cartesian product of the two sets of teams: \( 6 \times 5 = 30 \) games.

36. The number of combinations is equivalent to the Cartesian product of \( \{ \text{SLACKS} \}, \{ \text{SHIRTS} \} \), and \( \{ \text{SWEATERS} \} \) so the number of elements is \( n(\{ \text{SLACKS} \}) \times n(\{ \text{SHIRTS} \}) \times n(\{ \text{SWEATERS} \}) = 1 \times 5 \times 5 = 60 \) combinations.

Review Problems

44. (a) There are 11 letters in "common sense" but \( o, m, n, s, \) and \( e \) are repeated, so there are 6 elements.

(b) There are nine letters in "committee" but \( m, t, \) and \( e \) are repeated, so there are 6 elements.

45. (a) The question is really, "how many subsets of \( \{2, 3, 4\} \) are there?" \( \rightarrow 2^3 = 8 \) subsets not containing 1.

(b) Of the \( 2^4 = 16 \) subsets of \( A \) there are 8 not containing 1, so there must be 8 subsets which contain 1.

(c) All except \( \emptyset \), \( \{3\} \), \( \{4\} \), \( \{3, 4\} \). Thus \( 16 - 4 = 12 \) subsets containing 1 or 2.

(d) \( 16 - 12 = 4 \) subsets.

(e) See (a) and (b) above:

(i) There are \( 2^4 = 16 \) subsets not containing the element 5.

(ii) There are \( 2^0 = 2^4 - 16 = 16 \) subsets containing the element 5.

(f) Keep all subsets of \( A \), then list again adding the element 5 so the number of subsets will double to \( 32 \).

46. (a) \( \mathcal{B} = \{ x \mid x = 2n + 2, n = 0, 1, 2, 3, \ldots \} = \{2, 4, 6, 8, 10, \ldots \} = A \)

(b) \( \mathcal{C} = \{ x \mid x = 4n, n \in N \} = \{4, 8, 12, 16, \ldots \} \Rightarrow \mathcal{C} \subseteq A \) and \( \mathcal{C} \subseteq B \)